STEP-BY-STEP SOLUTION TO LISTENER 4295, "CODEBREAKER"

A neat feature of Listener 4295 is that, although it's a very difficult crossnumber puzzle, it can be solved through a series of simple, logical steps — if you can find them. All you really need is a list of square numbers up to three digits and a pocket calculator, and the amount of calculating you have to do is not excessive. Only at the very end, to fill the last eight cells of the grid, did I need to resort to "brute force" methods, going methodically through all the remaining possibilities one at a time to see which ones were impossible and which one would lead to a full solution. (It may still be, of course, that there is a way to get all the way to the end by simple steps, and I just haven't found it yet.)

It seemed clear right away that I needed some kind of simple notation for distinguishing the encoded digits entered in the grid from the unencoded digits they stood for. Otherwise I was going to get very confused very quickly. I decided on circling the original digits and leaving the grid entries uncircled. That's not so convenient for typing, so for these notes I'm going to put the original digits in parentheses and italics. I'll give the same treatment to clue references, which are in italics and parentheses when referring to the actual answer to the clue, plain when referring to the grid entry. So I can notate the clue for 1 Across in this way: (*1A*) = 20A.

For the first step of this solution, I'll use a and b to represent two unknown digits. Each one will be italicized and in parentheses when it represents a digit in a clue answer, and plain when it represents an encoded digit in a grid entry.

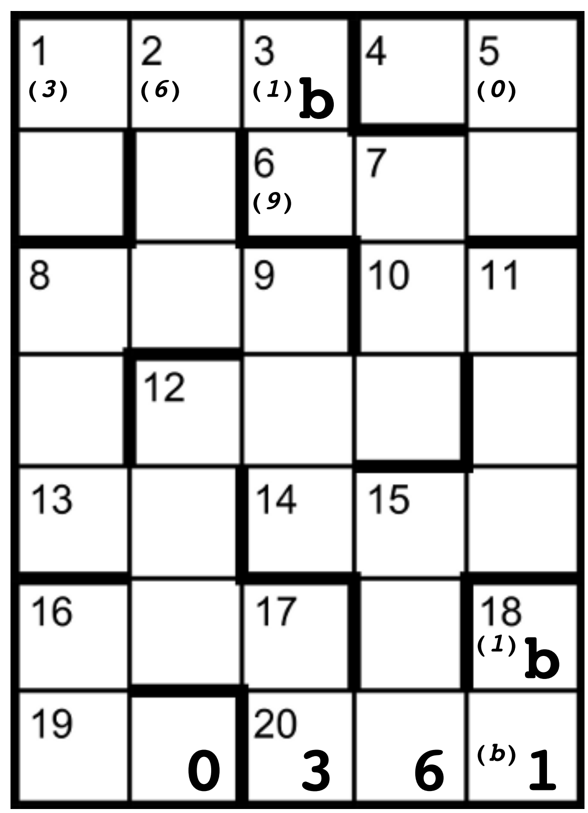
(*18D*) = 18D reversed. Let's say (*18D*) = (*ab*); then 18D is ba. Then (*a*) is encoded by b, and (*b*) is encoded by a. The last digit of 20A is thus (*b*) encoded as a. a and b are not the same digit, as no digit is encoded to itself.

(*1A*) = 20A. The last digit of (*1A*) is thus (*a*), which we know is encoded by b.

(*3D*) = the square root of 20A. The first digit of (*3D*) is (*a*), and the last digit of 20A is a, so we're looking for a two-digit number whose first digit is the same as the last digit of its three-digit square. The only two possible combinations are 11/121 and 19/361, but 20A can't be 121 because all ten digits entered in the top and bottom rows are different (from the instructions). So (*3D*) is (*19*), and 20A is 361.

Because the ten digits entered in the top and bottom rows must all be different, one of them must be zero; but no grid entry can start with a zero, so 0 must be the last digit of 19A. (*4A*) equals 19A, so the last digit of (*4A*) is (*0*).

At this point in the solution, the grid looks as shown at right. When I was solving the puzzle, I got to this point fairly quickly, then hit a wall for a long while. I was also able to find the first digits of 8A and 12D and the last digit of (*9D*) without much trouble, but those bits of information don't lead anywhere yet, so I'm not including them in this solution till later. It took me quite a bit of head-scratching before I found the next real step forward.



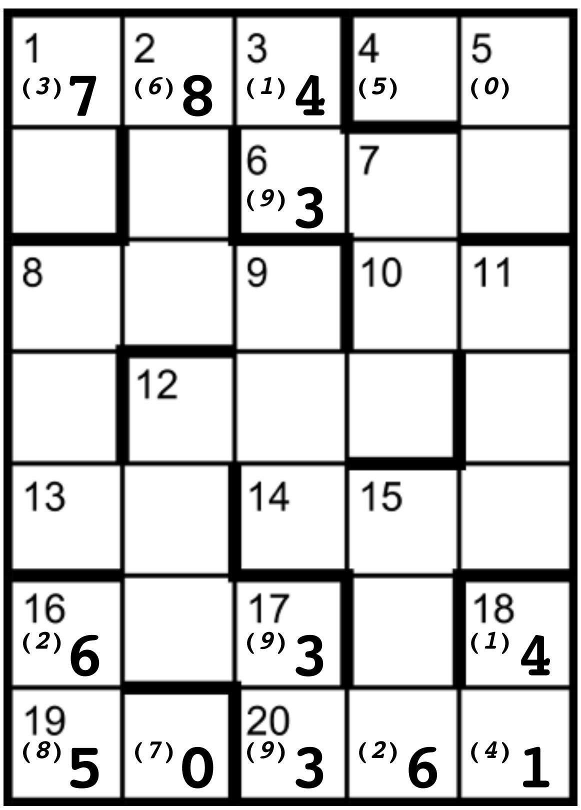
(*16D*)^2 = 1A, so 1A is a three-digit square number with no repeated digits and in which no digit is 0, 1, 3, or 6 (as the ten digits in the top and bottom rows are all different, from the instructions). Furthermore, the last digit of 1A cannot be 9, because if b were 9, the grid entries at 3D and 18D would both be 91, which is impossible (from the instructions). The only three-digit square that meets all these conditions is 784. So 1A = 784, and (*16D*) = (*28*).

(Not realizing that the last digit of 1A couldn't be 9 was what was holding me up; for a long time I was thinking that 289 and 729 were also possibilities.)

(*20A*) is a multiple of 17D. (*20A*) is a three-digit number ending in (*4*); its middle digit is even (from the clue to 15D), but not (*0*), (*4*), or (*6*), because the ten digits entered in the top and bottom rows must be different (from the introduction); so the last two digits of (*20A*) are either (*24*) or (*84*). Also because the ten digits in the top and bottom rows must be different, the first digit of (*20A*) cannot be (*0*), (*1*), (*3*), (*4*), or (*6*), and (*20A*) cannot be (*224*) or (*884*). (*20A*), then, must be one of (*284*), (*524*), (*584*), (*724*), (*784*), (*824*), (*924*), and (*984*). Only one of these is divisible by any two-digit number ending in 3, so (*20A*) is (*924*), and 17D is 33. Now we know six pairs of original and coded digits, so we can fill in a few more cells in the grid.

The first digit of (*4A*) is the first digit of 19A (from the clue to 4A); as this digit cannot already be entered in the top or bottom rows, either as an original digit or as an encoded grid entry, this digit can only be 5. As the ten original digits in the top and bottom rows must all be different, the last digit of (*19A*) can only be (*7*).

The grid now looks like this:



$(1D)^2$ = 12D, so (*1D*) is either (*30*) or (*31*) and 12D is either 900 or 961. The last digit of 12D is either 0 or 1, so the middle digit of (*16A*) is either (*7*) or (*4*), which means that (*16A*) is either (*279*) or (*249*). (*16A*) is a multiple of 5D, a two-digit number whose first digit by elimination is either 2 or 9. (*249*) is not a multiple of any such number, so (*16A*) must be (*279*), and 5D can only be 93. By elimination, the first digit of 4A is 2, so we now know how all ten digits are encoded. 12D is 900, and (*1D*) is (*30*).

(*6A*) = 900 plus a square; (*6A*) begins and ends with (*9*), and its middle digit can't be (*0*) or else (*7D*) would start with a zero (which is impossible from the introduction), so (*6A*) can only be (*949*).

(*9D*) = 12A – 7D. 12A begins with 9, and 7D begins with 1, so (*9D*) begins with either (*7*) or (*8*). But if (*7D*) began with (*7*), the encoded grid entry for 7D would begin with 0, so this is impossible and (*7D*) begins with (*8*). As 12A and 7D end with the same digit, (*9D*) must also end with (*0*).

(*13A*) = 8A – 8D; (*13A*) ends with (*7*) and 8A ends with 5, so 8D ends with 8. (*2D*) = 8A + 8D, so (*2D*) ends with (*3*), and 8A and 8D start with 3. Now we know that 8A = 375 and (*13A*) = (*67*), so 8D = 375 – 67 = 308. (*2D*) = 375 + 308 = (*683*).

The grid now looks like this:

1 (3) 7	2 (6) 8	3 (1) 4	4 (5) 2	5 (0) 9
(0) 9	(8) 5	6 (9) 3	7 (4) 1	(9) 3
8 (9) 3	(3) 7	9 (8) 5	10	11
(7) 0	12 (0) 9			
13 (6) 8	(7) 0	14 (0) 9	15	
16 (2) 6	(7) 0	17 (9) 3		18 (1) 4
19 (8) 5	(7) 0	20 (9) 3	(2) 6	(4) 1

At this point we know that the sum of the digits in the grid entries cannot reach 200, so the first digit of (*11D*) is (*1*). I don't have a step-by-step method to progress any further than that. I filled the last seven cells by brute force, going through all the possibilities for the second digit of 12A and seeing where each one led. Nine of the ten digits led sooner or later to impossibilities; only 4 led to a solution. My final solution is below.

1 (3) 7	2 (6) 8	3 (1) 4	4 (5) 2	5 (0) 9
(0) 9	(8) 5	6 (9) 3	7 (4) 1	(9) 3
8 (9) 3	(3) 7	9 (8) 5	10 (9) 3	11 (1) 4
(7) 0	12 (0) 9	(1) 4	(6) 8	(5) 2
13 (6) 8	(7) 0	14 (0) 9	15 (2) 6	(1) 4
16 (2) 6	(7) 0	17 (9) 3	(7) 0	18 (1) 4
19 (8) 5	(7) 0	20 (9) 3	(2) 6	(4) 1

Many thanks to Zag for a beautifully constructed puzzle!

— David Scott Marley, 25 May 2014